SIMULATION TESTING OF THE SABBATICAL MODEL ESTIMATOR FOR THE ASSESSMENT OF SOUTHERN HEMISPHERE HUMPBACK WHALE BREEDING STOCK C AND ITS COMPONENT SUB-STOCKS

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ABSTRACT

This paper develops various operating models (OMs) for the C1 and C3 substocks of humpback whales in the western Indian Ocean which allow interchange between the two. These operating models are used to assess the performance of the ***Sabbatical*** estimator. Generally the 90% probability intervals for the estimates from the ***Sabbatical*** estimator cover the true values for the OMs, though there is a tendency to underestimate r and consequently overestimate K. If the OM (but not the estimator) is sex-disaggregated, actual abundances for C1 can sometimes fall below and those for C3 sometimes above the 90% probability intervals for the estimator. Importantly if the true value of the interchange rate parameter is fixed to be considerably higher than values estimated from the present data, the estimates rates are also higher, and both pre-exploitation and current estimates of abundance are lower.

KEYWORDS: HUMPBACK WHALES, SIMULATION TESTING, INTERCHANGE

INTRODUCTION

Johnston and Butterworth (2009) implement four models (***Resident***, ***Sabbatical, Tourist*** **and** ***Migrant***) to estimate parameters for the C1 and C3 substocks, including the probability of interchange between them, using a Bayesian approach which takes account of capture-recapture information from photo-id data. This estimator generally captures the underlying parameter values reasonably, though with a tendency to estimate r too low and K too high. Interchange rates are also reasonably estimated, both when the true rates are low and high. In the latter case, abundance estimates in terms are lower.

Here a range of Operating Models (OMs) are defined and used to test the ***Sabbatical*** estimator.

METHODS

The Operating Models

The following OMs are considered in the simulation testing of the ***Sabbatical*** estimator:

i) the resident OM

ii) the sabbatical OM

iii) the tourist OM

iv) the migrant OM

The full model specifications for the OMs above are found in detail in Johnston and Butterworth (2009).

A further four OMs are also considered, all of which are based on the sabbatical model:

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v) the sex-disaggregated OM – see the appendix for modelling details. The sex-disaggregated catch histories are described in Muller *et al.* (2009). Here the mark recapture data are generated accounting for the sex-structure of the population, but these data are pooled over both sexes to use as input to the (sex-aggregated) estimator;

vi) the density dependence type 1 OM – see sensitivity test 5a in Johnston and Butterworth (2009), where density dependence acts on the sum of the abundance of the two stocks;

vii) the density dependence type 2 OM – see sensitivity test 5b in Johnston and Butterworth (2009), where density dependence acts on numbers present on the breeding grounds rather than breeding substock numbers; and

viii) the interchange rates for both sub-stocks are fixed at $\alpha = 0.3$. [Note that for this OM the interchange photo-ID data are excluded from the likelihood.]

Each OM was conditioned on the available data. Given that Bayesian estimation under sampling-importance-resampling (SIR) is conducted by first generating 500 000 realizations of the joint prior distribution, and then computing the likelihood for each sample from the prior, generation of data from each OM is based on the parameter vector with the highest likelihood, as a good approximation to the maximum likelihood estimate.

Each of these OMs was used to generate 100 pseudo-datasets for simulation testing purposes.

Data generation

For each simulation of the application the estimation model, a pseudo-dataset is generated from the OM under consideration. This data set consists of the following elements, corresponding to the data used for the assessment conducted during the 2008 meeting of the Scientific Committee in Santiago (IWC 2009):

1. 2003 Survey abundance estimate for C1 breeding grounds

$$\eta_{2003}^{C1,obs,sim} = \eta_{survey,2003}^{C1,true} e^{\varepsilon^{survey,sim}} \quad \text{where } \varepsilon^{survey,sim} \sim N(0,0.17^2)$$

$\eta_{survey,2003}^{C1,obs,sim}$ is the simulated data value for the C1 survey estimate of abundance in 2003 for simulation *sim*, and

$\eta_{survey,2003}^{C1,true}$ is the “true” value of the abundance of humpback whales on the C1 breeding grounds in 2003 obtained from the OM.

The CV of 0.17 assumed for the survey sampling variability is the estimate for the original survey (Johnston and Butterworth, 2009).

2. Cape Vidal SPUE for C1 breeding grounds

$$I_{SPUE,Vidal,y}^{C1,obs,sim} = I_{SPUE,Vidal,y}^{C1,true} e^{\varepsilon_y^{Vidal,sim}} \qquad \text{where } \varepsilon_y^{Vidal,sim} \sim N(0,0.27^2)$$

$I_{SPUE,Vidal,y}^{C1,obs,sim}$ is the simulated data value for the Cape Vidal SPUE in year *y* for simulation *sim*, and

$I_{SPUE,Vidal,y}^{C1,true}$ is the “true” value for the Cape Vidal SPUE value in year *y* obtained from the OM by assuming equality to the abundance present in C1 at that time (as this is used as a relative index, specifying the constant of proportionality as 1 does not matter).

The years *y* here are the years in which these surveys viz. 1988, 1989, 1990, 1991 and 2002 actually took place. The CV of 0.27 for these SPUE indices corresponds to the standard deviation estimate (corrected for bias) of the residuals about a log-linear regression fit of the original estimates against year.

3. Aircraft SPUE for C1

The true expected number of whale sightings in year y is known from the OM (see Equation (13) of Johnston and Butterworth (2009)):

$$\hat{\bar{n}}_y^S = q_{SPUE,aircraft} \eta_y^{C1,true} E_y$$

The probability of observing $\bar{n}_y^S$ as follows:

$$p(\bar{n}_y^S) = (\hat{\bar{n}}_y^S)^{\bar{n}_y^S} e^{-\hat{\bar{n}}_y^S} / (\bar{n}_y^S)!$$

where $\bar{n}_y^S$ = 0, 1, 2,…20+ (probability above 20 being negligible in practice and therefore lumped).

To generate the simulated data set of $\bar{n}_y^{S,sim}$ one first draws a random value Z from U[0,1].

The cumulative probability for each n is $\vec{p}(n) = \sum_{k=0}^{k=n} p(k)$ is then calculated.

Finally, the realised $\bar{n}_y^{S,sim}$ is given by:

IF Z < $\vec{p}(0)$ then $\bar{n}_y^{S,sim}$ =0

IF $\vec{p}(k-1) \le Z < \vec{p}(k)$ then $\bar{n}_y^{S,sim}$ =k.

4. Capture-recapture data for C1 and C3

First, the probability of seeing an animal in a particular breeding ground and year is considered. These values are fixed across all simulations and are calculated as:

$$p_y^i = \frac{n_y^i}{\eta_y^{i,true}}$$

where

p_y^i is the probability of seeing an animal in area i in year y for (which is the same for all simulations),

n_y^i is the number of animals successfully photographed in region i in year y (which is the same for all simulations and equal to the number of sighted in reality), and

$\eta_y^{i,true}$ is the "true" number of animals in area i in year y in terms of the OM.

The $\hat{m}_{y,y'}^{i,j}$ values then follow from the OM and are the same for all simulations. The probability of observing $m_{y,y'}^{i,j}$ is then calculated as follows:

$$p(m_{y,y'}^{i,j}) = \frac{\hat{m}_{y,y'}^{i,j\; m_{y,y'}^{i,j}}}{m_{y,y'}^{i,j}!} e^{-\hat{m}_{y,yp}^{i,j}}$$

where m = 0, 1 , 2,…11+ (probability above 11 being negligible in practice, and therefore lumped and truncated as 11).

To generate the simulated data set of $m_{y,y'}^{i,j,sim}$ one first draws a random value Z from U[0,1].

The cumulative probability for each m is $\vec{p}(m) = \sum_{k=0}^{k=m} p(k)$ is then calculated.

Finally, the realised $m_{y,y'}^{i,j,sim}$ is given by:

IF $Z < \vec{p}(0)$ then $m_{y,y'}^{i,j,sim} = 0$

IF $\vec{p}(k-1) \leq Z < \vec{p}(k)$ then $m_{y,y'}^{i,j,sim} = k$.

The estimator

The estimator examined here is the ***Sabbatical*** estimator. This estimator is implemented as described in Johnston and Butterworth (2009).

Simulation testing procedure

Each estimator is applied to the 100 generated datasets using the Bayesian methodology described in Johnston and Butterworth (2009). For each simulated dataset, the posterior median values of parameters of interest are stored. These are then finally summarised (across all 100 datasets) by calculating the medians of the 100 values for each such parameter. The results are reported in Tables 1a-g and compared to the OM "true" values. Tables 2a-g reports the RMSE (root mean square error) values of these posterior medians taken to provide the estimates of the quantities of interest.

RESULTS

Results for the ***Sabbatical*** model estimator when applied to data generated by each of the OM variants are reported in Tables 1a-h. Tables 2a-h report the RMSE (root mean square error) values.

DISCUSSION

For the ***Sabbatical*** model estimating from data generated on that basis, or based on any of the other baseline OMs (Tables 1a-d), there are no clear biases, though there is a tendency throughout for *r* to be estimated too low and (consequently) *K* too high. The 90% probability intervals span the true values for the interchange rate parameter α, or come close to zero when the resident model applies. The results for the alternate OMs for density dependence are similar in those respects (Tables 1f-g).

Importantly when the true interchange rate is high (α=0.3, Table 1h) the estimated α values are also high and consequently both initial and current abundances are estimated lower. However for the sex-disaggregated OM, the true C1 abundances are at times below and those for C3 at times above the 90% probability intervals for estimation under the sex-aggregated ***Sabbatical*** model (Table 1e).

When the OM is also the ***Sabbatical*** model, RMSEs for some parameters tend to be lower for C3 than for the other OMs, but C1 does not evidence a similar pattern.

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Table 1a: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **resident OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.098	0.088	0.079 [0.038; 0.098]	0.065 [0.027; 0.088]
K	8251	10348	8609 [7745; 11372]	12574 [9659; 17340]
α	0	0	0.028 [0.002; 0.116]	0.008 [0.001; 0.040]
N_{lowest}	240	2777	403 [262; 1349]	4433 [1521; 8876]
N_{2006}	6375	10347	6334 [5176; 7299]	12242 [9236; 16457]
N_{2006}/K	0.879	0.999	0.732 [0.512; 0.896]	0.999 [0.808; 1.000]

Table 1b: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **sabbatical OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.104	0.064	0.074 [0.037; 0.101]	0.066 [0.028; 0.089]
K	7698	11012	8513 [7216; 11408]	13639 [10211; 18621]
α	0.023	0.015	0.047 [0.004; 0.148]	0.017 [0.001; 0.064]
N_{lowest}	293	2578	553 [283; 1920]	5441 [2189; 10157]
N_{2006}	7234	10935	7197 [5834; 8435]	13305 [9957; 17873]
N_{2006}/K	0.939	0.993	0.855 [0.608; 0.972]	0.999 [0.842; 1.000]

Table 1c: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **tourist OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.105	0.096	0.074 [0.037; 0.101]	0.066 [0.028; 0.089]
K	7919	9306	9028 [7834; 11921]	11675 [9344; 15637]
α	0.019	0.008	0.036 [0.003; 0.122]	0.018 [0.002; 0.070]
N_{lowest}	254	1769	705 [325; 2093]	3543 [1312; 7023]
N_{2006}	7170	9305	7495 [6299; 8833]	11204 [8864; 14600]
N_{2006}/K	0.905	0.999	0.879 [0.631; 0.981]	0.998 [0.779; 1.00]

Table 1d: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **migrant OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.074	0.089	0.078 [0.042; 0.102]	0.066 [0.029; 0.089]
K	7499	9931	8628 [711; 13503]	11909 [9499; 16185]
α	0.019	0.015	0.046 [0.004; 0.146]	0.020 [0.002; 0.071]
N_{lowest}	277	1261	606 [299; 1678]	3669 [1425; 7228]
N_{2006}	6978	9768	7313 [584; 8438]	11520 [9066; 14990]
N_{2006}/K	0.931	0.984	0.877 [0.639; 0.980]	0.998 [0.785; 1.000]

Table 1e: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **sex-disaggregated OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.078	0.076	0.65 [0.021; 0.101]	0.066 [0.022; 0.089]
K	4824	16935	8607 [5349; 15010]	11711 [8957; 17546]
α	0.076	0.031	0.098 [0.009; 0.253]	0.058 [0.007; 0.137]
N_{lowest}	341	3087	1003 [346; 2967]	3574 [1507; 6895]
N_{2006}	4762	16880	6061 [4135; 8183]	11217 [8495; 14271]
N_{2006}/K	0.987	0.997	0.744 [0.373; 1.000]	0.998 [0.627; 1.000]

Table 1f: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **density-dependence type 1 OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.099	0.016	0.076 [0.035; 0.101]	0.066 [0.029; 0.089]
K	9696	13158	8676 [7152; 12077]	10129 [8459; 14082]
α	0.032	0.007	0.037 [0.004; 0.128]	0.025 [0.002; 0.089]
N_{lowest}	352	4613	751 [311; 2319]	1716 [590; 4234]
N_{2006}	9095	7719	7411 [6157; 8906]	9081 [7280; 11388]
N_{2006}/K	0.938	0.587	0.867 [0.618; 0.980]	0.949 [0.654; 1.000]

Table 1g: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to the **density-dependence type 2 OM** generated data.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.105	0.096	0.075 [0.039; 0.102]	0.066 [0.029; 0.089]
K	7850	9357	8565 [7341; 11608]	11521 [9231; 15521]
α	0.019	0.007	0.038 [0.004; 0.128]	0.018 [0.002; 0.069]
N_{lowest}	314	1843	594 [301; 1834]	3385 [1161; 6877]
N_{2006}	7528	9301	7342 [6030; 8535]	11091 [8784; 14438]
N_{2006}/K	0.959	0.994	0.863 [0.629; 0.973]	0.997 [0.772; 1.000]

Table 1h: ***Sabbatical*** model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to generated data from the **OM** where both interchange rates are fixed at $\alpha = 0.3$.

	"True" Values from OM		***Sabbatical* Model estimator**	
	C1	**C3**	**C1**	**C3**
r	0.105	0.082	0.061 [0.021; 0.101]	0.068 [0.033; 0.087]
K	5395	8259	6340 [2798; 14153]	9867 [5330; 14900]
α	0.3	0.3	0.216 [0.027; 0.359]	0.278 [0.098; 0.380]
N_{lowest}	937	389	1024 [379; 2655]	1393 [440; 2834]
N_{2006}	5393	6995	4399 [1773; 9119]	8711 [3795; 11893]
N_{2006}/K	1.000	0.855	0.868 [0.248; 1.000]	0.918 [0.531; 1.000]

Table 2a: RMSE values of the ***Sabbatical*** estimator when fitted to **resident-OM** generated data.

	C1	**C3**
r	0.023	0.022
K	1069	3014
α	0.031	0.009
N_{lowest}	270	2673
N_{2006}	543	2782
N_{2006}/K	0.176	0.016

Table 2b: RMSE values of the ***Sabbatical*** estimator when fitted to **sabbatical-OM** generated data.

	C1	**C3**
r	0.033	0.002
K	1642	3231
α	0.037	0.008
N_{lowest}	611	3551
N_{2006}	545	3083
N_{2006}/K	0.114	0.006

Table 2c: RMSE values of the ***Sabbatical*** estimator when fitted to **tourist-OM** generated data.

	C1	**C3**
r	0.036	0.030
K	1715	2741
α	0.026	0.026
N_{lowest}	852	2310
N_{2006}	787	2418
N_{2006}/K	0.067	0.016

Table 2d: RMSE values of the ***Sabbatical*** estimator when fitted to **migrant-OM** generated data.

	C1	**C3**
r	0.011	0.023
K	1465	2448
α N_{lowest}	0.037 510	0.011 2879
N_{2006}	598	2275
N_{2006}/K	0.094	0.015

Table 2e: RMSE values of the ***Sabbatical*** estimator when fitted to the **sex-disaggregated-OM** generated data.

	C1	**C3**
r	0.051	0.032
K	3939	3188
α N_{lowest}	0.041 697	0.039 1457
N_{2006}	896	2692
N_{2006}/K	0.262	0.023

Table 2f: RMSE values of the ***Sabbatical*** estimator when fitted to **density-dependence type 1-OM** generated data.

	C1	**C3**
r	0.031	0.049
K	1296	3025
α N_{lowest}	0.018 926	0.023 2849
N_{2006}	1693	1728
N_{2006}/K	0.096	0.355

Table 2g: RMSE values of the ***Sabbatical*** estimator when fitted to **density-dependence type 2-OM** generated data.

	C1	**C3**
r	0.033	0.030
K	1594	2653
α	0.028	0.014
N_{lowest}	646	2176
N_{2006}	594	2372
N_{2006}/K	0.119	0.019

Table 2h: RMSE values of the ***Sabbatical*** estimator when fitted to generated data from the OM where both interchange rates are fixed at $\alpha = 0.3$.

	C1	**C3**
r	0.049	0.016
K	3674	2104
α	0.096	0.059
N_{lowest}	706	1182
N_{2006}	1614	2060
N_{2006}/K	0.316	0.115

Appendix: Sex disaggregated population model

The age- and sex-aggregated population model which has been used for modeling the dynamics of the Southern Hemisphere humpback whales in recent assessments is as follows:

$$N_{y+1} = N_y + rN_y\left(1-\left(\frac{N_y}{K}\right)^{2.39}\right) - C_y \tag{A1}$$

The extension of the above model to incorporate sex-structure is to replace Equation (1) by:

$$N^m_{y+1} = N^m_y + rN^f_y\left(1-\left(\frac{N_y}{K}\right)^{2.39}\right) - C^m_y \tag{A2}$$

$$N^f_{y+1} = N^f_y + rN^f_y\left(1-\left(\frac{N_y}{K}\right)^{2.39}\right) - C^f_y \tag{A3}$$

where:

N_y is the total number of whales at the start of year *y*, which is given by $N_y = N^m_y + N^f_y$,

N^m_y is the total number of male whales at the start of year *y*,

N^f_y is the total number of female whales at the start of year *y*,

K is the carrying capacity,

C^m_y is the number of male whales caught in year *y*, and

C^f_y is the number of female whales caught in year *y*.

Equations (A2) and (A3) require past catches to be differentiated by sex (see Muller *et al.* 2009).